Topics in Communication Complexity

Assignment # 1

Due: May 5, 2026, 11:59pm

You may work on this together or individually. Please cite any people that you worked with (from the class) as well as any other sources. You will be graded on your logic and ideas for solving the technical questions below (3 and 4), as well as the clarity of your writeup (so getting the problem correct is not necessary or sufficient for a good solution.)

1. If you haven't already done so, please prepare lecture notes to accompany your presentation for the topic where you were the Main Presenter. Please email me the pdf file as well as the latex file. A template for the notes is on the course webpage.

2. Pick one topic that was presented in class where you did not have a role in the presentation. In your own words, please give a short review of the paper and presentation (one page). Here are some suggested points:

   (a) a brief summary of the paper(s) being presented.

   (b) Pros: an explanation of what you liked about the topic/result.

   (c) Cons: What were the limitations of the result?

   (d) Feedback: What did you like about the presentation? Give one or two suggestions for improvement in the presentation.

3. $GT(x, y)$, the greater-than function, is 1 if and only if $x > y$ (viewing x and y as numbers expressed in binary, each as n-bit numbers).

   (a) Prove that any deterministic protocol for GT requires cost $\Omega(n)$.

   (b) Give an upper bound of $O(\log^2 n)$ on the randomized complexity.

4. Consider the following two-player promise problem, $PromiseEQ(x, y, y')$. Alice's input is an n bit binary string x and Bob's input is y, y', $y \neq y'$, $|y| = |y'| = n$ with the promise that either (1) $y = x$ or (2) $y' = x$. Decide which case (1) or (2) holds.

   (a) Given an $O(1)$ public-coin randomized communication protocol for this problem.

   (b) Prove that $\Omega(\log n)$ bits of communication is necessary for deterministic protocols. (In fact, $\Omega(\log n)$ bits of communication are even required for nondeterministic protocols. You will receive extra credit for proving this stronger result.)

5. Discuss a research problem (in at most one page) that you are interested in and that you think may be connected to communication complexity, or information theory more broadly. This is an open-ended problem to give you a chance to explore your own research topic and to try to make connections with communication or information complexity.